Unravelling Mathematical Practices and Social Geometry from and With a Fishing Community*

(Des)enredando as práticas matemáticas e a geometria social da e com a comunidade piscatória

(Des)enredando prácticas matemáticas y geometría social desde y con una comunidad de pescadores

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Abstract

The aim of this article is to analyze the mathematical practices and the social geometry of profit distribution in a fishers' community of the *Arte-Xávega* in Portugal. The research is carried out by combining critical ethnography with communitarian education, in two learning spaces, one belonging to the fishing community, and the other, to the academic community. Special emphasis will be placed on the participatory nature of the data collection processes, especially on the analysis of information. The discussion will show how mathematical practices affect and are determined by social relations within the fishing community in a continuous ongoing process. it will also show how the leading role of Mathematics (education) in the search for a more just and equitable society. The research makes a small contribution to the reflection on the implications of Ethnomathematics for Mathematics (education) as a tool to change socioeconomic reality.

Keywords

local-traditional knowledge; communitarian education; social geometry; socioeconomic inequality

Resumo

A nossa intenção é analisar as práticas matemáticas e a geometria social da distribuição nos lucros de uma comunidade piscatória da Arte-Xávega em Portugal. A investigação realiza-se combinando a etnografia crítica com a educação da comunidade, em dois espaços de aprendizagem, um pertencente à comunidade pesqueira e outro à comunidade académica. Será dada especial ênfase à natureza participativa dos processos de recolha e, em especial, de análise da informação. O debate evidenciará como as práticas matemáticas afectam e são determinadas pelas relações sociais no seio da comunidade piscatória, num processo em constante evolução, bem como o papel protagonista da matemática (educação) em busca de uma sociedade mais justa. A investigação contribui como um grão de areia para a reflexão sobre as implicações da Etnomatemática na educação matemática como instrumento de transformação da realidade sócioeconômica.

Palavras-chave

conhecimento local-tradicional; educação comunitária; geometria social; desigualdade socioeconómica

Resumen

El propósito de este artículo es analizar las prácticas matemáticas y la geometría social de la repartición de la ganancia en una comunidad de pescadores del *Arte-Xávega* de Portugal. La investigación se realiza combinando la etnografía crítica con la educación comunitaria en dos espacios de aprendizaje: uno de la comunidad pescadora y otro de la comunidad académica. Se pondrá especial énfasis en el carácter participativo de los procesos de recolección y, sobre todo, del análisis de la información. La discusión nos permitirá evidenciar cómo las prácticas matemáticas inciden y, a su vez, son determinadas por las relaciones sociales dentro de la comunidad de pescadores en un proceso en continua evolución. Asimismo, se vislumbrará el rol protagónico de la (educación) matemática en la búsqueda de una sociedad más justa. La investigación aporta su granito de arena a la reflexión sobre las implicaciones de la Etnomatemática para la educación matemática como instrumento de transformación de la realidad socioeconómica.

Palabras clave

conocimiento local-tradicional; educación comunitaria; geometría social; desigualdad socioeconómica

Introduction

The Ethnomathematics Programme (D'Ambrosio, 2008, 2012) brings together researchers interested in developing diverse mathematical ideas, concepts, and actions by different cultural groups, to building a holistic view of mathematical knowledge, thus perceiving the influence of these concepts and actions in human social construction. Nonetheless, the motivations of interest are different depending on the context, so they have different purposes. Therefore, the views and ways to approach research reflect the diversity of contexts and are continuously evolving to adapt to the socio-cultural reality (Albanese et al., 2017). Today, many ethnomathematics researchers are working to find answers to questions such as: what are the implications of the Ethnomathematics Programme for (mathematics) Education? Touching on more general questions such as: how to integrate the needs of increasingly multicultural contexts with globalisation positions? (D'Ambrosio, 2006), or, however, in a neo-colonialist context, for whom should ethnomathematics research be positioned?

The current agenda of the Ethnomathematics Programme contributes to the construction of a fairer society (D'Ambrosio, 2018) through the reallocation of the human intellectual construction in comparison with the different primary concepts of survival—such as counting, measuring, inferring, analysing, etcetera—identified in Western culture (Restivo, 1994) as mathematical knowledge. This construction is based on respect for the dignity of life and valuing the encounters of diversity (D'Ambrosio, 2008). One of the current challenges of the Ethnomathematics Programme is to formulate alternatives to the formal mathematics education system towards an education that transforms society (D'Ambrosio, 2012).

Communitarian education (Mesquita et al., 2014, p. 12) is a concrete possibility in this regard. It already exists within a community (Coppe & Mesquita, 2015), as it is the process that enables that community to establish a shared knowledge base for survival. This conception of education has its roots in Paulo Freire's *popular education* (1970), which proposed, through the emancipation of local knowledge, different from that imposed in formal education, the construction of a political consciousness to fight against the culture of silence (Freire, 1970). The concept of the culture of silence and its corollary, the politics of silencing, correspond to a set of ways of being, thinking and expressing oneself that are the consequence of a social structure of secular domination in Latin America, which are also manifested in developing countries, and in the marginalised communities of countries considered developed, conditioning both oppressors and oppressed. The culture of silence is one of the dimensions of the banking concept of education (Freire, 1970), which is revealed in the low voice and lack of communication that characterise a society without dialogue.

A critical research practice, when focused on communitarian education, encourages community members to become aware of the educational processes intrinsic to their survival, concretising communitarian education as a praxis of living in the community with respect for its spatiotemporal diversity. This process enables researchers to be with and within the community, to know the environment from the community's own perspective and thus to identify local knowledge collaboratively with its members.

The local knowledge that emerges is transdisciplinary, in the sense that D'Ambrosio (2008) gives to the concept of ethnomathematics[1], as the ways and techniques (*-tics*) of understanding, explaining and dealing with (*matema-*) the diverse natural, cultural and socio-economic dimensions of reality (*ethno-*). The learning of this knowledge is based on a form of organisation, here understood as communitarian. The trivium curriculum proposes to organise the learning process around three instruments: literacy (communicative instruments), matheracy (analytical instruments), and technoracy (material instruments).

In their work, Mesquita et al. (2011) include a discussion of the three concepts provided by D'Ambrosio (2008) in the trivium curriculum, based on local learning practice. For the authors, the concept of literacy—which, according to D'Ambrosio (2008), is the ability to process information, such as the use of written and spoken language, signs and gestures, codes and numbers—points to a new meaning of reading. Literacy requires, for example, reading a film, social media content, or even reading political actions, governance proposals and the social relations surrounding the reader's context. Nowadays, reading also includes skills of interpretation of graphs, charts and other forms of informing the individual, thus including comprehension and expression with language condensed into codes.

According to the same authors, *materacy*, which is defined and understood by D'Ambrosio (2008) as the ability to infer, propose hypotheses and draw conclusions from data, is a first step towards an intellectual stance, invisible in our school systems… Materacy is closer to the way mathematics was present in both classical Greek and indigenous cultures. The focus was not simply on counting and measuring, but on divination and philosophy. Materacy entails a deeper reflection on man/woman and society and should not be restricted to the elite, as it was in the past.

Following Mesquita et al. (2011), *technoracy* is the critical familiarity with techniques and technologies. Technology, here, is not linked to computer technology development, but to each instrument that can collaborate in

1 We clarify that the *Ethnomathematics Programme* (capital and singular) refers to the research programme as detailed in previous paragraphs. While we use *ethnomathematics* (minuscule and plural) to refer to the object of study of the Programme, as indicated in this paragraph.

the materialisation of actions on human relations. Today, with technology focusing more on the digital than on the analogue, the operational aspects are, in most cases, inaccessible to a large part of the still excluded population.

Communitarian (mathematics) education combined with Critical Ethnography (Gérin-Lajoie, 2009; Thomas, 1993) establishes a bottom-up approach that foresees the creation of spaces for interaction between people with diverse experiences and life stories to collectively discuss their problems and define their own way of acting in society. The integration of approaches and points of view provides solutions that improve society as a whole and can be re-proposed, with appropriate adaptations, in other contexts.

This critical and participatory approach is a possible solution to some of the tensions in the Ethnomathematics Programme (Pais, 2010; Parra-Sánchez, 2017) that problematise the relationships established between researcher and researchee, and between ethnomathematical currencies, reproducing colonial power schemes. Other researchers have proposed different ways of confronting these tensions sharing the same direction, i.e., the search for more symmetry in this relationship: the investigation of the practice itself (Mesquita et al., 2011), the mutual interrogation (Adam et al., 2010), the dichotomy between recognition and the search for (ethno) mathematics (Albanese et al., 2017), the notion of *Propio* as a decolonial tool (Parra-Sánchez, 2018) and the dialogical perspective as a balance between the emic and the etic (Rosa & Orey, 2018).

Research approach

Among the various research interest focuses, we propose here to analyse the practice of profit sharing by fishermen and its relationship with mathematics and social geometry from a critical and participatory methodology based on community education.

In the following, we will briefly review the main concepts that we will need later on for our analysis. On the one hand, we will describe the mathematical notions involved in this practice from an educational perspective. On the other hand, we will clarify, based on previous definitions, what we mean by social geometry.

The sharing of profit between different people is a situation of sharing which—among many others (measurement, quotient, division) (Llinares & Sánchez, 1988; Mancera, 1992; Pinxten et al., 1983)— gives meaning to the mathematical concepts of *fraction* and *ratio*. There are different types of distribution: the partitioning of the whole or part/whole situation occurs when a certain number of congruent parts are made from a whole or total —which we will henceforth indicate as equal in order to approach the

common language used by fishermen—; from these, some parts are taken, that constitute a *portion* of the total. Uniform sharing, also called equitable sharing, establishes that a whole/total is shared out so that each partition or portion is equivalent in number (if the whole is discrete) or quantity (if it is a continuous whole) to another partition or portion. Proportional sharing occurs when a whole is shared out in such a way that each partition or portion maintains a certain relationship with the others, which can be additive or multiplicative (Godino, 2004).

Now, the mathematical representation of each portion of the division is usually made with a fraction *a/b*, where the denominator, *b*, indicates the number of parts into which the whole is divided and the numerator, *a*, indicates the number of parts that make up a portion or partition. However, to indicate the multiplicative relationship that exists between two portions when the division is proportional, we use the ratio, which we indicate here with the notation *c:d*, which means that for every *c* parts that make up a portion, the other is made up of *d* parts (Godino, 2004).

The term *social geometry* was coined at the end of the 19th century by the German sociologist George Simmel, who studied social relations, processes and boundaries as fundamental categories of sociological thought. This is the origin of a movement among sociologists who approached the study of social space by investigating problems related to social movement and social dimensions using a geometric-mathematical language (Ferreira, 1955).

In more recent times, in the architectural field, the term *social geometry* has been used to indicate the geometry of the social interaction of people, which conditions the shape of the spaces to be built or constructed (Fernández-Álvarez, 2006).

Le Bras (2000) conceives social geometry as the study of the mathematical models that govern the distribution in space of peoples and individuals. This process is guided by two opposing forces —attractive and repulsive— which have caused humankind (as communities or as individuals) to follow tendencies to disperse and tendencies to associate throughout the ages, reaching alternating stages of equilibrium and crisis.

It is the latter that has inspired our own conception. In the context of the present ethnomathematical study, we are interested in analysing social geometry, in the sense of how mathematical practices reflect social relations and their materialisation in the social space within the fishing community and, vice versa, how social relations and their positioning in the social space influence the mathematical practices of the community.

Furthermore, we will see how these relationships are transformed by technological innovation that revolutionises the practice of fishing, which also strongly conditions mathematical practices.

Methodology: critical ethnography

The methodology used places special emphasis on the participation of the community in the processes of data collection and analysis, which are carried out through critical ethnography (Gérin-Lajoie, 2009; Thomas, 1993), due to the flexibility that this provides when opening up to dialogue with the community.

This methodology allows the integration of different cultural constructs of the group under study with the observer's vision of the culture, to accompany the processes of identity production, in which research becomes an immersion of the researcher in the experience of the chosen group and vice versa. This process implies the participation of key actors of the group under study in the dynamics of the academy, such as data collection, report production, data analysis and dissemination of the results, in this case through the teaching work.

This process involves the integration of communitarian education (Mesquita et al., 2014) as a research methodology. However, in this case it has been more of an inter-communitarian education, where a two-way dialogue has been generated between the local fishing community and the academic community.

All of this makes it possible to problematise the educational and social challenges *with* the community, blurring the roles of researcher/researchee, and allowing for a critical analysis of the academic community's approach to education and its limitations in collaborating with the current issues of the dichotomy between local and global needs (Mesquita, 2017).

Learning spaces

Spaces of encounter (Mesquita, 2016; Mesquita et al., 2011), the fulcrum of communitarian education, are key elements for the dynamics of dialogue. In this research, we distinguished two moments during which the encounters took place in different spaces. Firstly, one of the researchers was immersed in the spaces of the fishing community of the Costa de Caparica - Portugal, in the winter of 2017-2018. This was made possible by the creation of the Ocean Literacy Observatory[2] (OLO) research laboratory, which brings together members of the academy, technicians, and local communities. Secondly, the participation of fishermen in academic activities has been organised, so that their discourse becomes the starting point for the reflection of a class on Ethnomathematics in a master's degree at the University of Granada for two consecutive years at the beginning of 2019 and 2020.

2 www.olo.blue/

Figure 1. *In the learning space constituted by the* alveolos *of the fisherman Mr. Mário Raimundo*
Source: Photo taken during the immersion in the fishing community, September 2017.

It is worth noting that this community of fishermen had already worked closely with the OLO coordinator in previous projects: *D.A.R. à Costa – Tr@nsFormArte* and Urban Boundaries[3] – in building a mutual interest in the major issues of development and sustainability of local artisanal fisheries, through meetings in countless spaces of encounter over the last 18 years. Here, a bond of belonging to local causes, both fishing and academic, was created, thus building up a unified body of emancipatory, libertarian, and counter-colonialist actions. All this facilitated the entry and active participation of the fishermen in the two moments of this research, fostering their interest and desire to participate into this research and its context, to be with a new researcher, overcoming, through a social and slow pedagogy, also the barriers of linguistic diversity; to be with master's students in another country, share knowledge and get to know other visions on artisanal fishing in different parts of the world; after all, the master's students came not only from Spain, but also from Venezuela, Colombia, and Brazil.

During the first moment, we went daily to the *alveolos*, a building near the Costa de Caparica (Portugal) beach where each fisherman is assigned a small enclosed space to store his tools and work on the nets (Figure 1), and to the beach (Figure 2), to carry out non-participant observation, participant observation (the researcher came to help the fishermen in their work of repairing nets), and individual in-depth unstructured interviews with several fishermen (Geertz, 1973) about working with the nets and about fishing. All this information has been compiled in the field notes of one of the researchers.

3 http://fronteirasurbanas.ie.ul.pt/

Figure 2. *In the learning space constituted by the beach, observing the repair of nets by Mr. Mário Pedro*

Source: Photo taken during the immersion in the fishing community, November, 2017.

Finally, the OLO Room, still in Costa de Caparica, is a space conceived and created by the OLO study and research group and financed by the MARE Research Centre, and is located in a building near the alveoli where the fishermen keep their tools and repair their nets, and in the so-called fishermen's neighbourhood, where they live. It is precisely because it is so close to the area where the fishermen live and work that the encounter dynamics and opportunities for dialogue are greatly facilitated.

This room has been the setting for the collective elaboration with the fishermen of an ethnographic report (Angrosino, 2012) that arose from the sharing with three fishermen of the field notes resulting from the encounters described above. These three fishermen were chosen by the fishing community itself to represent it, as they are in charge of the two active associations of traditional fishing arts that exist nowadays on the Costa da Caparica.

Such analysis arises from the proposal to take the systematisation of academic work to the collective practice within the community, since in the critical ethnographic process all those involved discuss the field notes in their meetings, point by point.

The report is the result of a collective and shared reflexive analysis during which the main role of the researchers has been to organise the information, while the fishermen commented among themselves, clarifying and thinking together about the implications of the visibility of the issues raised for understanding the establishment of social —and power— relations within their own community, in other words, analysing the data themselves (Figure 3). Importantly, this process has enabled the triangulation

of information from different sources and lends communal validity (Moral, 2006, p. 156) not only to the information collected, but also to the analysis and interpretation of the data itself, which is a characteristic of critical participatory ethnography.

Figure 3. *In the learning space constituted by the olo room, working together on the ethnographic report with Mr. Mário Raimundo and Mr. Lídio*
Source: Photo taken during the immersion in the fishing community, January 2018.

The dynamics of encounter in a critical ethnography is based on the cyclical socialisation between different communities, in this case the academic and the fishing communities, which have a common interest. In this study, the academic community aims to uncover the cloak of invisibility that hangs over the local-traditional knowledge of the fishing community, which has requested collaboration in order for its voice to be heard in local political decisions. Here, the purpose was to create a more participatory space in the local society, to guarantee some of their basic cultural traditions for their survival, such as the making of the fishing net and the knowledge of fish production —preparing to fish, fishing and working economically on their fish.

In this sense, the dynamics proposed in the encounters was based on dialogue and knowledge transfer from the communities to which they belong, in order to co-construct a common local knowledge —the starting point for the building up of a bottom-up collaborative work.

In the second moment, the academic space consisted of a university classroom of a master's class at a Spanish university where the fishermen organised a workshop on net construction (Figure 4), bringing their work tools to class and teaching the students how to weave a net cloth. What interests us most for the subsequent discussion, they gave a presentation on the history of the fishing community of the Costa de Caparica and

the distribution of profit among the fishermen of a company, taking into account its variations over the last decades. The meeting with the groups of master's students was a new opportunity for the fishermen themselves to re-elaborate their knowledge and the implications of mathematical practices for the social fabric of their cultural group. The activity was videotaped for the participation of some virtual students in non-synchronous mode.

Figure 4. *In the learning space constituted by the classroom of the master's class taught by the fishermen, showing the nets to the students*
Source: Screenshot taken from the recording, January 2020.

In this second moment, the academic community received the fishing community —the opposite movement to the first moment. This exchange promotes intercultural dialogic interaction, which makes it possible to make visible and value the knowledge of the different communities that participate in the encounters. In addition, the fishing community manages to elaborate a translation of its own knowledge systematised and expressed in a language accessible to the academic community.

These intercultural intellectual encounters allowed to re-signify the processes of community mathematics education that objectify justice. This praxis exemplifies a position of the Ethnomathematics Programme in which the promotion of the encounter, recognition, valorisation, and dialogic integration of any knowledge is valued, building a new common, efficient, effective, and relevant knowledge for the solution of local symptoms of global problems. Thus, the Ethnomathematics Programme aims to understand and work on the relational positioning of cultural knowledge, how it is constructed and how it evolves, which shows the degree of justice between the parties involved in the learning processes.

From these spaces of encounter, we selected fragments that allude to the way in which the fishermen communicate their knowledge about the reading of their reality (literacy), analyse the relationships that are established in the social space in real life and their decisions and actions (matheracy), as well as the specific techniques —tools and strategies, here mainly of calculation— created to express and materialise their knowledge in their work (technoracy). Below, we present a possible analysis through these concepts, remembering that they are transversal and are identified here in a specific context, always bearing in mind their close relationship.

Selected data of interest on profit sharing

Although *Arte-Xávega*[4] *is legally considered a trawl fishing art, in practice it is carried out with an encircling net to pull ashore, sometimes referred to as a cod-end seine. This means that the Arte-Xávega* net is thrown into the sea from a boat, leaving one end of the network cable to land and returning the end of the other cape to land, which then allows the net to be pulled ashore —formerly by hand, nowadays by using tractors— with the fish. The fishermen gather in companies, that is, groups of people who fish together with the same boat and net. Each fisher performs certain tasks and his/her financial reward for his/her work depends on the type of tasks he/she performs.

Below, we have selected, from the data collected throughout the research —i.e., from the ethnographic report co-constructed with the fishermen and from the transcription of the master's session organised by the fishermen, the most exemplary fragments describing the distribution of profit in some fishing companies at different historical moments, using the fishermen's own way of communicating it (literacy—their own reading of reality expressed through a specific language).

> Extract 1
>
> In the 50's and 60's, the sharing of the profit among the fishermen of a company was as follows: from what was earned from the sale of the fish, first the expenses were taken out, which were then the oil for the lanterns, the taxes and the *faena* (this is the money charged for the task of setting the net on the boat before going fishing, which the *calador* did with two other men and earned extra money for it), and it is divided into four parts. One part, i.e., a quarter of the profit, goes to the company owner. The remaining three parts are divided as follows:

4 In Spain, there is evidence of a similar traditional fishing art. In Malaga, Andalusia, it takes the name of *Jábega*. Here, we wanted to keep its original name in Portuguese. We did the same with other technical fishing terms and expressions that we left in literal translation in English and will be highlighted in italics.

» three parts for the sea *mestre do mar* (who drives the boat),

» two and a half parts for the *espadilhero* (who handles a long oar used as a rudder),

» two and a half parts for the *calador* (the one who throws the net from the boat),

» two and a half parts for the *mestre do terra* (the one who sets and fixes the net),

» two parts for all the rowers on the boat (they vary in number, but in the 50's and 60's they were between 16 and 8, depending on the size of the boat and the weather conditions),

» a part and a quarter for the paddlers left on shore,

» a part for the shore fishermen (who always stay on shore, and there are many of them), of whom there are 3 who are paid a quarter more for their respective tasks: the wood man (who was in charge of the wooden tools for carrying the net and slipping the boat), the lantern man, and the one who calls the company together.

» Half a part for the women.

» A quarter of a part for the children, usually they are three or four, who are in charge of emptying the boat of water and sand and winding up the ropes.

All of them are also in charge of hauling in the net.

» A quarter of a part went to the company hut, where the temporary fishermen stayed overnight.

The unsold fish was divided into equal shares for everyone, a little pile for each one, and in the few cases where there was a difference between species, a lot was drawn.

(From the report made jointly with the fishermen, February 2018).

It should be noted that the company's scribe is in charge of selling the fish and then making the accounts for the distribution of the profit.

Extract 2

Mr. Lídio: The scribe knew how many fishermen went on the boat that day, how many stayed on land, and how many children *went to Arte-Xávega* and he was in charge of *doing the accounts*. (Transcription and own translation of Mr. Lídio's explanation in the master's class, January, 2020)

An example of profit sharing in a company in the 70's-80's, when a single tractor was used to move the ship, is now presented.

Extract 3

At that time, the total profit, excluding expenses, i.e., petrol for the tractor, the engine and oil for the lanterns, was divided as follows:

» three parts for the *trafo* (i.e., the maintenance of the boat and the net)

» two parts for the tractor (there was only one tractor)

» one part for the boat's engine

» two parts for the *mestre do mar* and the rowers (i.e., all those who went out with the boat and the one who was in charge of assembling and repairing the net)

» one part for the people left on land (including women and children).

The unsold fish was always divided equally among all.

(From the report made jointly with the fishermen, February 2018).

Today's profit sharing reflects the amount of expenses they have to incur for the operation and maintenance of tractors and the ship's engine.

Extract 4

From the profit, after deducting the tax that is kept by the *lota*[5], the fuel for the ship's engine and tractors (which is sometimes counted in tractor working hours) is deducted. After that, the distribution depends on the company.

Each tractor takes more or less 5 parts (the tractors are usually 3 or 4: two tractors to pull the net, one for the boat and one to take the fish to the *lota*, being in some companies these last two tasks carried out by the same tractor).

The *trafo* (maintenance of engine, boat and net) takes another 5 parts, or sometimes 4 parts for the boat and net and 4 parts for the engine.

The men who go on the boat get more (2 or 3 or 4 parts, skippers usually get one more part) than those who stay on land (1 or 2 parts, those who drive the tractors may get a little more than the others on land), depending also on the constancy with which they *go to the beach* [i.e. participate in the fishing tasks].

Nowadays the work is not as physically hard as it used to be; the hardest tasks are to pick out the fish when the net comes ashore, and to set up and repair the nets.

(From the report made jointly with the fishermen, February 2018).

5 Portuguese term for the place where fishermen auction fish wholesale.

Results and participatory discussion. . .

During the research process the fishermen became researchers and analysed their own practices (here, the profit sharing) from a mathematical point of view and, at the same time, addressing the connections with the social geometry of their community. Below, we share the discussion over these questions, combining our observations with fragments of the discourses, which emerged from the different moments of collective analysis with the fishermen.

. . .about mathematics

First, it should be noted that the distribution of unsold fish among the fishermen of a company is always carried out according to an equitable distribution, since all the piles are equal and each fisherman is allocated only one. The distribution of the profit, on the other hand, follows a proportional distribution model. In fact, the total profit is divided once more into equal parts (as indicated by the fishermen), but each fisherman is allocated a certain number of parts according to his role in the fishing process.

The distribution model is based on an apparent simplicity and clarity, but it hides a certain complexity at the mathematical level —and more so in Extract 1, where multiplicative relations between ratios are involved, as we will see below. In the fishermen's discourses, they themselves recognise that these accounts are not for everyone. They are difficult, and their complexity is even greater for the perception of the common fisherman of the 20th century, who did not even have a primary school education, as evidenced in the fisherman's own words in t Extract 5.

> Extract 5
>
> Mr. Lídio: Most of the fishermen were illiterate, the first scribe of my father's [company] was my older brother, with third grade.
>
> (Transcription and own translation of Lídio's explanation in the master's class, January 2020).

In fact, the real amount earned by each fisherman is deeply linked to the total number of *parts* into which the total amount of profit is divided, that is why in Extract 2 it is evident that one of the main tasks of the scribe is to keep track of how many people, in their different roles, *go to the Arte-Xávega* (literacy-reading of reality and its quantitative representation), that is, they contribute that day to the fishing. This information is essential to make the accounts (technoracy-calculation tool).From the mathematical point of view, in all the extracts, the information relative to the ratio between the profit of one fisherman and the other is made public

(for example, one gets 1 part and the other 2, the ratio is 1:2), but the information relative to "the portion of the total profit that each one gets" is hidden. If we were to express the portion using a fraction to represent it, this is equivalent to knowing the numerator, but not the denominator. The process of calculating the denominator is quite complex because it is highly dependent on how many fishermen have participated in the fishing, and this is a variable data.

> Extract 6
>
> On one occasion, a fisherman did not want to go fishing in a company any more, because another owner offered him a part and a quarter instead of a single part. But on the day that the other owner earned little more than the first owner, the fisherman's portion was still less than what the fishermen of the first company earned. Textual words "*mais partes são feitas, mais pequena fica a unidade*" [the more parts are made, the smaller the unit becomes].
>
> (From the researcher's field diary, reflection by Mr. Mário Pedro, November, 2017).

The fisher-researcher highlights how receiving more parts does not mean that a fisher's profit (the portion of the total) increases by conducting an analysis of sharing practices (matheracy-analysis of sharing practice and its effect on profit). In the case described in Extract 6, by changing the company, the number of fishermen or the number of parts that each one earns could change, which would imply a partitioning of the total into a larger number of parts. In his final comment, the same fisherman-researcher underlines the existence of the inverse relationship between the size of a part and the number of parts by which the total profit is divided.

This information is key to realise, for example, the imbalance that would exist in Extract 1 between the owner's profit and that of another fisherman. In fact, what corresponds to the owner of the company is a quarter of the total profit (this is known as a portion of the total), while the rest is divided among the other fishermen without explicitly mentioning what fraction of the three quarters of the total (this is where, from the point of view of mathematics, a multiplicative relation between fractions is presented) corresponds to each part. In fact, here one can only deduce what is the ratio between the profit of one fisherman and another (excluding the owner).

The deep understanding of the mathematical concepts of ratio and portion (analysed in mathematical terms in previous sections), empowers those fishermen who become aware of what possible manipulations of information allow. Extract 7 provides evidence in this sense, being another element of matheracy as an analysis of the effects on profit of the application of certain sharing strategies.

Extract 7

Mr. Mário Pedro tells that once some fishermen of a company claimed that they wanted to earn more. So, the owner of the company had an idea, he agreed to allocate half a part more to each one, but he also allocated one more part to his "*espadilhero*" and to himself. Mr. Mário Pedro reflects that these fishermen were happy with the deal, not understanding that it all depends on the number of parts in which you divide the total.

(From the researcher's field diary, reflection by Mr. Mário Pedro, November, 2017).

The fishermen themselves, in our collective analysis, note again how the increase in the parts allocated to each fisherman does not imply an increase in profit. It may, in principle, appear to be an increase in profit, concealing the fact that, when the share rises, one actually loses out.

Indeed, mathematically, at the basis of this proposal are the principles of the concept of proportion and equivalent fractions. Extract 7 refers to the profit sharing of the 70's-80's explained in Extract 2, where we recall that land fishermen got one part and sea fishermen got two. The proposed increase allocates one and a half parts to land fishermen and three parts to sea fishermen, respecting the ratio 1:2 (equivalent to 1.5:3) between the profit of both.

Extract 8

Mr. Lídio: To get these accounts modified and make the fishermen earn some more money, I was a shipowner, skipper, and scribe. It was a way not to lose parts, to save money. (Transcription and own translation of Lídio's explanation in the Master's class, January, 2020)

In Extract 8, related to the 70's-80's, when the sharing was carried out as described in Extract 3, a strategy used by Mr. Lídio as owner and shipowner to reduce the splitting of the total—that is, the number of parts into which the total profit is divided— and thus make the shares —and therefore, the fishermen's profit— bigger, is highlighted. It is a specific technique that allows fishermen to adapt for survival (technoracy-calculation strategy).

In the following section we show how social changes and the introduction of technological innovations strongly influence the sharing of profit among fishermen, which, in turn, mirrors the changing relationships among fishermen in a company.

. . .about social geometry through social and technological transformations

It should be noted, firstly, that the fish is shared out on the basis of a uniform distribution. This practice, which advocates equality among all, according to the fishermen's conversations, is determined by relations of equity among the members of the fishing community, who are recognised as having the same food needs.

On the contrary, with regard to the sharing of profit, we recognise a model that, in academic terms, is identified as capitalist, in which the benefits are distributed proportionally according to certain hierarchy[6]. In fact, in all the extracts, a direct relationship between responsibility/danger and profit is identified (matheracy-analysis of the relationship between the position in the work hierarchy and its effects on profit), as made explicit by the fishermen themselves. However, in Extract 1, referring to the earliest period, a greater difference between the fishermen and the owner seems to stand out. In fact, the fishermen, who were mostly illiterate, as seen above, were in the hands of the company scribe and trusted him, as described in the following extract.

> Extract 9
>
> Mr. Lídio: The fishermen relied a lot on the scribe. The scribe was their mentor in legal terms. In one way or another, he safeguarded the interests of the fishermen. He might not be a member from the family of the owner of the company, but he was a person he trusted. (Transcript and own translation of Lídio's explanation in the master's class, January, 2020)

Here mathematics is presented as a tool of power in the hands of the owners of the companies, represented by their scribes, who had an evidently higher level of education.

But the analysis of the sharing leads the fishermen themselves (matheracy-analysis of the relationship between profit and position in the work hierarchy), at least those with some education, to become aware of the inequality. this is the trigger to lead a process of empowerment of their class.

> Extract 10
>
> Mr. Lídio: I felt that my father [the owner] earned a lot while the other workers earned a pittance. [...] There was a need to change the accounts. (Transcript and own translation of Lídio's explanation in the master's class, January, 2020)

6 The owner of the machinery (tractors, boats and nets) earns a much more consistent share, while the fishermen are a salaried workforce.

In extract 10, the fisherman-researcher makes explicit his unease about a situation that in his opinion was unfair. At this moment, mathematical knowledge leads to make certain injustice visible and is an instrument to bring about change.

In fact, this demand was made with the fall of the dictatorial regime and the beginning of the struggle for workers' rights in the mid-70's, which was joined by fishermen who began to organise themselves to demand the right to health care and retirement, as the fishermen themselves recounted in the master's session.

Moreover, this injustice is more evident towards women, as they are allocated less profit parts than men (see Extract 1), even though they perform the same task of hauling the net ashore. However, later on, as can be seen in Extract 3, women are paid the same wage as men (technoracy-tool or calculation strategy that allows for a certain greater balance), as explained in Extract 11, and children are no longer involved in the work, possible evidence of a schooling that prevents them from participating in fishing activities.

Extract 11

> Mr. Lídio: I used to work for the emancipation of women, I employed in my company, my daughter, my daughter-in-law and 7 other women on land. [...] There were no differences [regarding earnings] between men and women. It was a very important conquest that we achieved. (Transcript and own translation of Lídio's explanation in the master's class, January, 2020).

It is interesting to note how, over time, the model becomes simpler and the redistribution of profit becomes fairer, more balanced. In Extract 3, for example, we can see that the owner no longer earns a quarter of the total, but a certain number of the same parts as the fishermen. To this, it must be added that each company is made up of fewer fishermen, as we have already seen in Extract 8, and there is less differentiation between tasks, which leads to greater equality between the fishermen.

These simplifications have allowed (all of) the fishermen to become more aware of the process (matheracy-analysis of the relationship between profit and position in the social space of work) and have led them to demand even better conditions at the social level and more clarity in the practice of sharing.

We recall that in Extract 1, according to the fishermen themselves, there is no mention of many expenses for the maintenance of the boat or the nets, which were also paid by the owner of the company.

On the contrary, in Extract 3, and even more in Extract 4, referring to more recent times, a considerable increase of parts of the profit dedicated to the expenses for the introduction of the engine of the boat and tractors is highlighted. These parts end up in the hands of the company owner, who owns all the tools and manages their maintenance as well as the expenses. In a certain way, the model gains in transparency, since the owner himself makes it known to everyone that a good portion of the profit, although managed by him, is actually destined to the maintenance of these technological innovations that have profoundly revolutionised artisanal fishing, making the work physically less hard.

Conclusions

In this research we wanted to share the mathematical practices of the fishing community of *Arte-Xávega* in Costa da Caparica (Portugal). We highlight the intellectual tools of the community of artisanal fishermen, as we approach, in the extracts presented, both the way in which fishermen read and communicate reality (literacy) and analyse (matheracy) the relationships between their social position and its effects on profit, and the way in which they use specific calculation techniques as tools and strategies (technoracy) to adapt and apply their knowledge to their work.

In this context, the search for and construction of spaces of encounter between the fishing community and the academic community in which to carry out a reconstruction of the local community education process, has allowed us to co-construct a dialogue that has made possible collective reflection on the mathematics involved in the practice of profit sharing, as well as on social geometry. The later, we recall once again, consists of the analysis of the relationships established in the social spaces that are expressed in some mathematical practice which, in turn, reflects the social fabric of a community.

We also highlight how this experience of critical ethnography combined with community education is part of a set of proposals that problematise the relations between researcher and researchee, aiming for greater symmetry in the power relations in the research process itself, each of them responding to the specific demands of the context in which it is adopted.

Collectively with the fishermen we managed to analyse how social transformations, among which we highlight access to education —particularly their mathematical empowerment— and technological innovations have altered the relationships between fishermen, as can be seen in a modification of the mathematical practice of profit sharing. Moreover, it has become clear, in a very concrete context, how mathematics plays a leading role in the negation or search for a fairer society. On the one hand,

mathematical knowledge is seen as a tool of power in the hands of the few who handle it (the owners and the scribes); on the other hand, greater awareness of mathematical techniques is taken as an instrument of empowerment for fishermen who demand a more equitable redistribution of profit and greater transparency when it comes to making expenses visible.

Finally, we emphasise how, through the experience and collaboration of fishermen, also in the educational field, we have left a seed of change that is projected into the future in the awareness of teachers (master's students), key actors in the educational field (Albanese et al., 2017). In fact, sharing this experience within the framework of a teacher training course has provided the occasion, not only for a deeper reflection of the fishermen, but also to generate an interesting debate on the role of formal and informal education in relation to cultural practices, particularly in mathematics. The possibility of addressing, in school and academic learning spaces, real mathematical situations faced by the community is a concrete proposal for incorporating community education, generally informal, into formal education.

Although we do not go into this aspect in depth here, we would still like to leave —by way of conclusion and as a call to continue valuing these processes— the comment of a student in which the value of mathematics (education) for social change can be glimpsed.

> Student: [The fishermen] gained political and social awareness, taking an active role in a new construction of cultural practices that promoted equality. New mathematical operations, new ways of measuring, comparing and sharing would bring about this change, in which mathematics appears as a tool to improve the quality of life and the dignity of human relations. (From the reflections of a student in the Ethnomathematics course, February, 2020)

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